MTH 234 Section 104 Exam 1

Name: ______________________

Signature: ______________________

Date: ______________________

Do not start this exam until instructed; you will have 90 minutes to finish the exam. No notes, books, calculators, phones or electronic devices are allowed on this exam. If you have a question, raise your hand; otherwise, there is no talking during the exam.

There are 14 problems on this exam on 6 pages, in addition to this cover page. The point values of each problem vary, but are listed in the questions.

Good luck!

From *SMBC*.

Fill in the Blanks Section. No work needed, and no partial credit available. [4+4+6+4+4+3=25]

1. (4 points) A vector normal to the plane $3x + y = 7z$ is $\vec{n} =$ ______________________.

2. (4 points) A surface is given by $F(x, y, z) = x^3 + 2yz + y^2$. The equation of the tangent plane at the point $(1, 0, 1)$ is given by ________________________________.

3. (2+2+2=6 points) A particle has acceleration $\vec{a}$, velocity $\vec{v}$ and position $\vec{r}$. You are given that

$$\vec{a}(t) = \vec{i} - 3\vec{j}$$
$$\vec{v}(0) = \vec{k}$$
$$\vec{r}(0) = 2\vec{j} + \vec{k}$$

   Find the following:

   (a) $\vec{v}(t) =$ ________________________________.

   (b) $\vec{r}(t) =$ ________________________________.

   (c) Does the particle go through the origin? __________

Extra Work Space.

4. (4 points) Suppose that $(3, 4)$ is a critical point for the surface $h(x, y)$, and say that

$$h_{xx}(3,4) = 6, \quad h_{yy} = 1, \quad h_{xy}(3,4) = -2$$

Choose one of the following:

(a) $(3, 4)$ is a local maximum of h.
(b) $(3, 4)$ is a local minimum of h.
(c) $(3, 4)$ is a saddle point of h.
(d) There is not enough information to determine this.

5. (4 points) The two legs of a right triangle are measured to be 2 cm and 4 cm with a possible error of at most 0.3 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle: ____________________ cm^2.

6. (3 points) Consider $\vec{a} = \langle 1, 0, 0\rangle$ and $\vec{b} = \langle 4, 5, -1\rangle$. Then the vector projection $\text{proj}_{\vec{a}}\,\vec{b}$ is ____________________.

Extra Work Space.

Standard Response Questions. Show all work to receive credit. [10 + 15 + 10 + 5 + 10 + 10 + 10 + 5 = 75]

7. (10 points) Find a value of a such that $u(x,t) = \sin(4t)\cos(ax)$ satisfies the differential equation $u_{tt} = 4u_{xx}$.

8. (5+10=15 points) Evaluate the following limits. If one or both does not exist, say so.

   (a)

$$\lim_{(x,y)\to(-1,3)} \frac{2xy}{x^2+y^2}$$

   (b)

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

9. (7+3=10 points) Consider $\vec{r}(t) = \langle 2\sin t, t, 2\cos t\rangle$.

   (a) Find the arc length function for $\vec{r}(t)$ starting from the point $(0, 0, 2)$.

   (b) Suppose you move 1 unit along $\vec{r}(t)$ in the positive direction. Where are you now?

10. (5 points) Find a vector normal to the plane passing through the points $P = (1, 2, 3)$, $Q = (1, 0, 0)$, and $R = (2, 2, 2)$.

11. (10 points) Find the linearization of $f(x, y) = 2 + \sqrt{1 + x + \sin y}$ at the point $(0, \pi)$.

12. (10 points) Find the partial derivative $\frac{\partial T}{\partial r}$ for

$$T = \frac{v}{u}, \quad u = \frac{2rq^2}{s^2}, \quad v = rs$$

Your final answer should include only the variables q, r, s.

13. (5+5=10 points) Consider the function $g(x, y, z) = x + \ln(yz)$.

    (a) Find ∇g at the point $(3, 1, 2)$.

    (b) Find the directional derivative of g at $(2, 1, 2)$ in the direction of $\vec{i} + \vec{k}$.

14. (5 points) A ball is thrown in the air at an angle of $45°$ and an initial speed of $10\sqrt{2}$ m/s. How far away does the ball hit the ground? (Ignore air resistance, and use the value of $g \approx 10$ m/s^2).